

	P.R.Government College (Autonomous) KAKINADA	Program&Semester III B.Sc. Mathematics Major (V Sem)			
Course Code MAT- 504 T	TITLE OF THE COURSE Number Theory & Problem Solving Sessions				
Teaching	Hours Allocated: 60 (Theory)	L	T	P	C
Pre-requisites:	Basic Mathematics Knowledge on Integration	3	-	-	3

Course Objectives:

This course will cover the particular mathematical functions that have more or less established names and notations due to their importance in mathematical analysis, functional analysis, geometry, physics, or other applications.

Course Outcomes:

On Completion of the course, the students will be able to-	
CO1	Understand the fundamental theorem of arithmetic
CO2	Understand Mobius function, Euler quotient function, The Mangoldt function, Liouville's function, The divisor functions and the generalized convolutions.
CO3	Understand Euler's summation formula, application to the distribution of lattice points and the applications to $\mu(n)$ and $\Lambda(n)$
CO4	Understand the concepts of congruencies, residue classes and complete residues systems.
CO5	Comprehend the concept of quadratic residues mod p and quadratic non residues mod p .

Course with focus on employability/entrepreneurship /Skill Development modules

Skill Development		Employability		Entrepreneurship	
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Syllabus:

Unit – 1: The Fundament Theorem of Arithmetic.

Introduction, Divisibility, Greatest common divisor, Prime numbers, The fundamental theorem of arithmetic, The series of reciprocals of the primes, The Euclidean algorithm, The greatest common divisor of more than two numbers.

Unit– 2: Arithmetical Functions And Dirichlet Multiplication

Introduction- The Mobius function $\mu(n)$ – The Euler quotient function $\phi(n)$ - A relation connecting ϕ and μ - A product formula for $\phi(n)$ - The Dirichlet product of arithmetical functions- Dirichlet inverses and the Mobius inversion formula- The Mangoldt function $\Lambda(n)$ - multiplicative functions- multiplicative functions and Dirichlet multiplication- The inverse of a completely multiplicative function-Liouville's function $\lambda(n)$ - The divisor functions $\sigma_\alpha(n)$.

Unit – 3: Averages Of Arithmetical Functions

Introduction- The big oh notation. Asymptotic equality of functions- Euler's summation formula- Some elementary asymptotic formulas-The average order of $d(n)$ - The average order of the divisor functions $\sigma_\alpha(n)$ - The average order of $\phi(n)$ - An application to the distribution of lattice points visible from the origin- The average order of $\mu(n)$ and $\Lambda(n)$ -The partial sums of a Dirichlet product- Applications to $\mu(n)$ and $\Lambda(n)$.

Unit – 4: Congruences

Definition and basic properties of congruences- Residue classes and complete residue systems- Linear congruences- Reduced residue systems and the Euler- Fermat theorem- Polynomial congruences modulo p . Lagrange's theorem- Applications of Lagrange's theorem- Simultaneous linear congruences. The Chinese remainder theorem- Applications of the Chinese remainder theorem.

Unit – 5: Quadratic Residues and the Quadratic Reciprocity Law

Quadratic Residues, Legendre's symbol and its properties, Evaluation of $(-1/p)$ and $(2/p)$, Gauss lemma, The Quadratic reciprocity law, Applications of the reciprocity law, The Jacobi Symbol, Gauss sums and the quadratic reciprocity law, the reciprocity law for quadratic Gauss sums, Another proof of the quadratic reciprocity law.

II. Reference Books:

Text Book

Introduction to Analytic Number Theory by T.M.Apostol, Springer Verlag-New York, Heidelberg-Berlin-1976.

Reference Books

1. Elementary Number Theory by G.A.Jones and J.M.Jones, , Springer
2. Elementary Number Theory by David, M. Burton, 2nd Edition UBS Publishers.
3. Number Theory by Hardy & Wright, Oxford Univ., Press.
4. Elements of the Theory of Numbers by Dence, J. B &Dence T.P, Academic Press

III. Co-Curricular Activities:

Seminar/ Quiz/ Assignments/ Applications of Numerical methods to Real life Problem /Problem Solving Sessions.

BLUE PRINT FOR QUESTION PAPER PATTERN.

Paper –Major XV : Number Theory & Problem Solving Sessions

UNIT	TOPIC	S.A.Q	E.Q	Marks Allotted
I	The Fundament Theorem of Arithmetic.	01	01	15
II	Arithmetical Functions And Dirichlet Multiplication	02	01	20
III	Averages Of Arithmetical Functions	01	01	15
IV	Congruences	01	02	25
V	Quadratic Residues and the Quadratic Reciprocity Law	02	01	20
Total		07	06	95

S.A.Q. = Short answer questions

(5 marks)

E.Q . = Essay questions

(10 marks)

Short answer questions

: 4 x 5 M = 20

Essay questions

: 3 x 10 M = 30

Total Marks

:

= 50

Pithapur Rajah's Government College (Autonomous), Kakinada
III Year B.Sc., Degree Examinations - V Semester
Mathematics Course: Major XV : Number Theory & Problem Solving Sessions
(Model Paper w.e.f. 2025-26)

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Time: 2Hrs

Max. Marks: 50

SECTION-A

Answer Any Three Questions, Selecting At Least One Question From Each Part

Part – A

3 X 10 = 30

1. Essay question from Unit - I.
2. Essay question from Unit – II
3. Essay question from Unit - III.

Part – B

4. Essay question from Unit - IV .
5. Essay question from Unit - IV.
6. Essay question from Unit - V .

SECTION-B

Answer any four questions

4 X 5 M = 20 M

7. Short answer question from Unit - I.
8. Short answer question from Unit - II.
9. Short answer question from Unit – II.
10. Short answer question from Unit – III.
11. Short answer question from Unit - IV.
12. Short answer question from unit – V.
13. Short answer question from Unit – V.

PITHAPUR RAJAH'S GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS

Question Bank for
PAPER-MAJOR XV : NUMBER THEORY

Short Answer Questions

UNIT-1: FUNDAMENTAL THEOREM OF ARITHMETIC

1. If n is an even positive integer, prove that $2^{2n}-1$ is divisible by 15.
2. Find the gcd of 117,45.
3. If $a=2210$, $b= 493$ then find (a, b) and hence $[a, b]$.
4. If p is a prime and $a, b \in \mathbb{Z}$, $p|ab$ then Prove that $p|a$ or $p|b$.
5. Express 2025 as a product of prime numbers.

UNIT-II: ARITHMETICAL FUNCTIONS AND DIRICHLET MULTIPLICATION

1. If $n \geq 1$ prove that $\sum_{d|n} \mu(d) = \begin{cases} 1 & \text{if } n = 1 \\ 0 & \text{if } n > 1 \end{cases}$.
2. Find $\tau(50000)$ and $\sigma(50000)$.
3. Find $\varphi(n)$ if $n = 5, 6, 8, 12, 15, 35, 60, 72, 100, 256$.
4. If $(m, n) = 1$ then prove that $\varphi(n, m) = \varphi(n) \varphi(m)$.
5. Mobius function, Euler -Totient function, Mangoldt function.
6. For all f , prove that $f * I = I * f = f$.

UNIT-III: AVERAGE OF ARITHMETICAL FUNCTIONS

1. Show that $\zeta(2) = \frac{\pi^2}{6}$.
2. Explain $\sum_{n=1}^{\infty} \frac{\mu(n)}{n^2} = \frac{6}{\pi^2} + O\left(\frac{1}{x}\right)$.
3. Prove that the Average Order of $\sigma_1(n)$ is $\frac{1}{2} \zeta(2)$.
4. Define oh notation and asymptotic equality of functions.

UNIT-IV: CONGRUENCES

1. If $a \equiv b \pmod{n}$ then Prove that $ac \equiv bc \pmod{n}$ for any $c > 0$.
2. Find the remainder obtained dividing the sum by 12, $1! + 2! + 3! + \dots + 99! + 100!$.
3. If $ac \equiv bc \pmod{n}$ then prove that $a \equiv b \pmod{n|d}$ where $d = \gcd(c, n)$.

4. If $(a,m)=1$ then prove that $a^{\phi(m)} \equiv 1 \pmod{m}$.

5. Solve the congruence $5x \equiv 3 \pmod{24}$.

UNIT-V: QUADRATIC RESIDUES AND THE RECIPROCITY LAW

1. Find the Legendre's symbol of $\left(\frac{77}{43}\right)$.

2. If p is odd prime then prove that $\left(\frac{2}{p}\right) = (-1)^{\frac{p^2-1}{8}}$.

3. Find the value of $\left(\frac{7}{11}\right), \left(\frac{22}{11}\right)$.

4. Find Jacobi symbol of $\left(\frac{109}{385}\right)$.

5. Find Jacobi symbol of $\left(\frac{5}{567}\right)$.

6. For a prime $p = 11$ find the quadratic residue $\pmod{11}$.

ESSAY QUESTIONS

UNIT-1: FUNDAMENTAL THEOREM OF ARITHMETIC

1. State and prove Division Algorithm.

2. State and prove Euclidean Algorithm

3. State and prove the relation between L.C.M and G.C.D.

4. State and prove the Fundamental Theorem of Arithmetic

UNIT-II: ARITHMETICAL FUNCTIONS AND DIRICHLET MULTIPLICATION

1. If $n \geq 1$ prove that $\varphi(n) = \sum_{d|n} \mu\left(\frac{n}{d}\right)$ (OR) $\frac{\varphi(n)}{n} = \sum_{d|n} \frac{\mu(d)}{d}$.

2. If $F(n) = \sum_{d|n} f(d)$ then $f(n) = \sum_{d|n} \mu(d) F\left(\frac{n}{d}\right) = \sum_{d|n} \mu\left(\frac{n}{d}\right) F(d)$.

3. If $n \geq 1$ prove that $\varphi(n) = n \prod_{p|n} \left(1 - \frac{1}{p}\right)$.

4. If $n \geq 1$ prove that $\sum_{d|n} \varphi(d) = n$.

UNIT-III: AVERAGE OF ARITHMETICAL FUNCTIONS

1. For any $x > 1$ Prove that $\sum_{n \leq x} \varphi(n) = \frac{3}{\pi^2} x^2 + O(x \log x)$.

2. For any $x \geq 1$ Prove that

$\sum_{n \leq x} d(n) = x \log x + (2c - 1)x + O(\sqrt{x})$, where c is Euler's constant.

3. For any $x \geq 1$ Prove that $\sum_{n \leq x} \frac{1}{n} = \log x + c + O\left(\frac{1}{x}\right)$.

4. For any $x \geq 1$ Prove that $\sum_{n \leq x} \sigma(n) = \frac{1}{12} \pi^2 x^2 + O(x \log x)$.

UNIT-IV: CONGRUENCES

1. The linear congruence $ax \equiv b \pmod{n}$ has solution iff $d \mid b$ where $d = \gcd(a, n)$.

2. What is the remainder when 2^{20} is divisible by 41.

3. If $(a, m) = 1$ the solution (unique mod m) of linear congruence $ax \equiv b \pmod{m}$ is given

by $x \equiv ba^{\phi(m)-1} \pmod{m}$.

4. State and prove the Lagrange's theorem.

5. State and prove Chinese Remainder Theorem.

6. For any prime p , prove that all the coefficients of polynomial $f(x) = (x-1)(x-2) \dots (x-p+1) + 1$ are divisible by p .

UNIT-V: QUADRATIC RESIDUES AND THE RECIPROCITY LAW

1. If p is an odd prime and $(a, p) = 1$ then prove that a is a quadratic residue of p

Iff $a^{\frac{p-1}{2}} \equiv 1 \pmod{p}$.

2. If p is an odd prime then prove that $\left(\frac{-1}{p}\right) = \begin{cases} 1 & \text{if } p \equiv 1 \pmod{4} \\ -1 & \text{if } p \equiv 3 \pmod{4} \end{cases}$.

3. State and prove Gauss Lemma.

4. If P and Q are odd positive integers with $(P, Q) = 1$ then prove that

$$\left(\frac{P}{Q}\right) \left(\frac{Q}{P}\right) = (-1)^{\frac{P-1}{2}} (-1)^{\frac{Q-1}{2}}.$$

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Teaching	Hours Allocated: 30 (Practicals)	L	T	P	C
Pre-requisites:	Basic Mathematics Knowledge on Integration	-	-	2	1

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4. Elements of the Theory of Numbers by Dence, J. B &Dence T.P, Academic Press

Semester – V End Practical Examinations

Scheme of Valuation for Practical's

Time : 2 Hours

Max.Marks : 50

- **Record** - 10 Marks
- **Viva voce** - 10 Marks
- **Test** - 30 Marks
- **Answer any 5 questions. At least 2 questions from each section. Each question carries 6 marks.**

BLUE PRINT FOR PRACTICAL PAPER PATTERN

COURSE-Major XV : Number Theory & Problem Solving Sessions

Unit	TOPIC	E.Q	Marks allotted to the Unit
I	The Fundament Theorem of Arithmetic.	2	12
II	Arithmetical Functions And Dirichlet Multiplication	1	12
III	Averages Of Arithmetical Functions	1	06
IV	Congruences	2	06
V	Quadratic Residues and the Quadratic Reciprocity Law	2	12
	Total	08	48

PITHAPUR RAJAH'S GOVT. COLLEGE (AUTONOMOUS), KAKINADA
III year B.Sc., Degree Examinations - V Semester
Mathematics Course-Major XV : Number Theory & Problem Solving Sessions
(w.e.f. 2023-24 Admitted Batch)
Practical Model Paper (w.e.f. 2025-2026)

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Time: 2Hrs

Max. Marks: 50M

Answer any 5 questions. At least 2 questions from each section.

5 x 6 = 30 Marks

SECTION - A

1. Unit – I
2. Unit – I
3. Unit – II
4. Unit - III

SECTION – B

5. Unit – IV
6. Unit – IV
7. Unit – V
8. Unit – V

➤ **Record** - 10 Marks

➤ **Viva voce** - 10 Marks
